ances of emitter and collector layers were 44 Ω/sq and 6·5 Ω/sq, respectively. As compared with MBE HBTs (fabricated with the same process),[4] MOVPE HBTs showed lower or comparable emitter, base and collector resistances.

The current gains of self-aligned MOVPE HBTs were found to be rather low. For large-area devices (emitter dimensions: $8 \times 20\,\mu m^2$), the device current gain was measured to be less than 10. The current gain of a representative microwave device with two emitter fingers (MTT2, emitter dimensions: $1{\cdot}2 \times 9 \times 2\,\mu m^2$) was measured around 5 as shown in Fig. 1*a* and 1*b*, respectively. However, because of low device parasitics the microwave performances of self-aligned MOVPE HBTs are very impressive regardless of the low DC current gain. Cascade Microtech probes were exploited during the microwave measurement to make contacts to the MOVPE HBTs.

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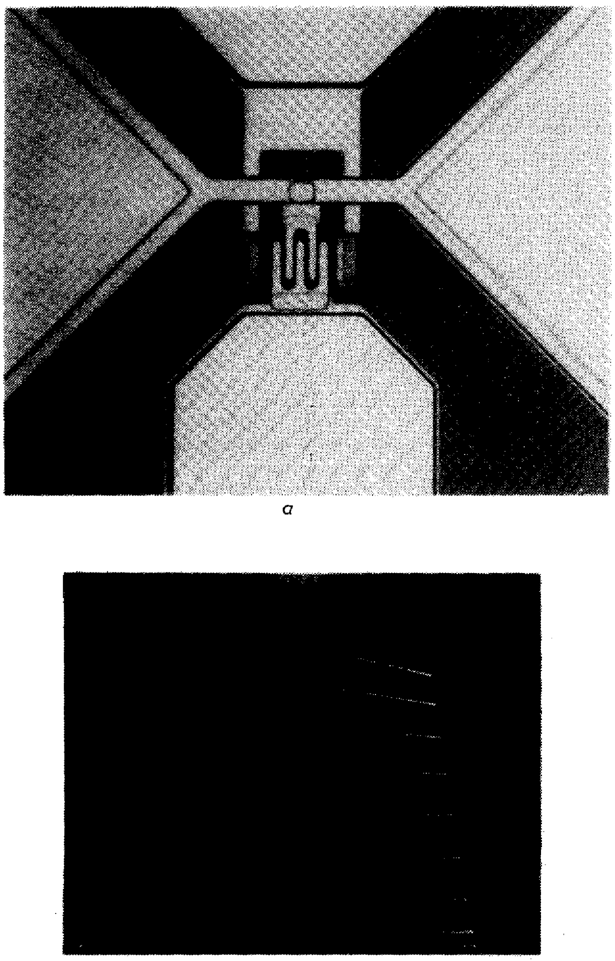

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Fig. 1

a Self-aligned MOVPE HBT with two 1·2 μm-wide emitters
b *I/V* characteristics of MOVPE HBT; measuring scales I_c (5 mA/div), I_b (1 mA/step) and V_{ce} (0·5 V/div)

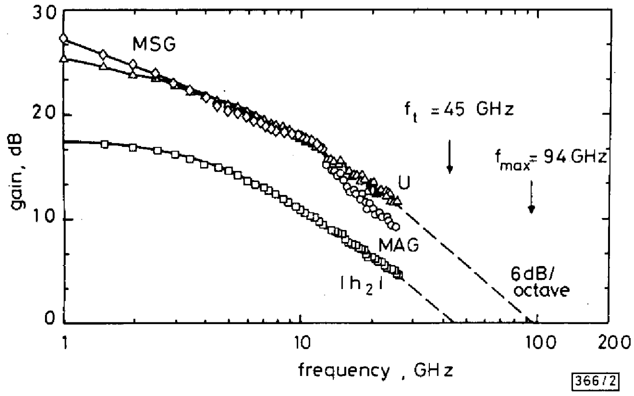

Fig. 2 *Unilateral gain G_u and current gain h_{21} derived from S-parameter measurements with Cascade Microtech probe testing*

Based on *S*-parameters measured up to 26 GHz, the extrapolated current gain bandwidth of an MOVPE HBT (MTT2) was 45 GHz and the maximum frequency of oscillation was 96 GHz, as shown in Fig. 2. The above device has a base-collector junction area of approximately $80\,\mu m^2$. During the test, the transistor was biased at $V_{ce} = 1{\cdot}6$ V with an emitter current density of $6{\cdot}5 \times 10^4$ A/cm^2.

Conclusion: A self-aligned AlGaAs/GaAs HBT with a base doping of 1×10^{20} cm^{-3}, emitter doping of 7×10^{17} cm^{-3} and n^+ contact layer dopings of 1×10^{19} cm^{-3} have been grown, fabricated and characterised. An f_{max} of 94 GHz and an f_t of 45 GHz have been achieved.

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METHOD OF REGULARISATION FOR INVERSE OF CONVOLUTION OF FINITE-LENGTH SEQUENCES

Indexing terms: Signal processing, Convolution, Matrix algebra, Identification

The problem of 'ill-posedness' for the case of the inverse of the convolution of finite-length discrete signals is examined. A method of regularisation that enables us to obtain a reasonable solution to such an inverse problem is introduced, and a new definition of the condition number is proposed. A simple example of application is presented.

Introduction: Inverse problems are very often ill-posed ones. Roughly speaking the 'ill-posedness' means that a small error in the input data causes a significantly large error at the output.[1] The methods which enables us to bound the output error, yielding thus a reasonable approximate solution, are generally termed the 'regularisation'.[1,2] The inverse of the convolution of the finite-length sequences discussed here is a typical example of an ill-posed problem. The aim of this letter is to propose a method of regularisation which, with certain restrictions, gives a reasonable, approximate solution to such a problem. In addition, a simple criterion, expressed in the form of the condition number, which enables us to choose the input signal so that the ill-conditioning is avoided, is proposed. Finally a simple example is presented.

Formulation of problem: Consider the problem which consists in the identification of the impulse response of a linear system on the basis of its input and output signal. Now, denote the following:

(*a*) input signal: $s = \{s_1, s_2, \ldots, s_k\}$
(*b*) output signal: $g = \{g_1, g_2, \ldots, g_r\}$

where r and k are the natural numbers and $r > k$.

Assuming that the signals g and s are given, we seek a solution for the system impulse response $h = \{h_1, h_2, \ldots, h_m\}$, $m = r - k + 1$. The above formulation is the implication of the following relationship:

$$s * h = g \tag{1}$$

where the asterisk denotes convolution.

Solution to problem: The operation of the convolution (eqn. 1) can be expressed in the following matrix notation:

$$r\left\{\overbrace{\begin{bmatrix} s_1 & 0 & \ldots & 0 \\ s_2 & s_1 & \ldots & 0 \\ \cdot & \cdot & & \vdots \\ \cdot & \cdot & & s_1 \\ \cdot & \cdot & & \vdots \\ s_k & & & s_2 \\ 0 & s_k & & \vdots \\ 0 & 0 & & \cdot \\ 0 & 0 & \ldots & s_k \end{bmatrix}}^{m}\right. \begin{bmatrix} h_1 \\ h_2 \\ \cdot \\ \cdot \\ \cdot \\ h_m \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ \cdot \\ \cdot \\ \cdot \\ g_r \end{bmatrix} \tag{2a}$$

or in contracted form:

$$\boldsymbol{S}_T \,.\, \boldsymbol{h}_m = \boldsymbol{g} \tag{2b}$$

where $\boldsymbol{S}_T$ is the input signal Toeplitz matrix, $\boldsymbol{h}_m$ is the impulse response sought and $\boldsymbol{g}$ is the output signal. From the viewpoint of the matrix algebra, the relationship of eqn. 2*a* is formally equivalent to the following:

$$r\left\{\overbrace{\begin{bmatrix} s_1 & 0 & \ldots & 0 & 0 & 0 & \ldots & 0 \\ s_2 & s_1 & \ldots & 0 & 0 & 0 & \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \vdots \\ \cdot & \cdot & \cdot & s_1 & 0 & 0 & \cdot & 0 \\ \cdot & \cdot & \cdot & \vdots & \vdots & \vdots & \cdot & \vdots \\ s_k & & \cdot & s_{k-m+1} & \sigma_1 & 0 & \ldots & 0 \\ 0 & s_k & \ldots & & 0 & \sigma_2 & \ldots & 0 \\ 0 & 0 & \ldots & & 0 & 0 & \ldots & 0 \\ 0 & 0 & \ldots & s_{2k-m-1} & 0 & 0 & \ldots & \sigma_{k-1} \end{bmatrix}}^{r}\right. \begin{bmatrix} h_1 \\ h_2 \\ \cdot \\ \cdot \\ \cdot \\ h_m \\ \vdots \\ h_r \end{bmatrix} = \begin{bmatrix} g_1 \\ g_2 \\ \\ \cdot \\ \cdot \\ \cdot \\ \\ g_r \end{bmatrix} \tag{3a}$$

or in contracted notation:

$$\boldsymbol{S} \,.\, \boldsymbol{h} = \boldsymbol{g} \tag{3b}$$

where $h_{m+1} = h_{m+2} = \ldots = h_r \equiv 0$, and $\sigma_1, \ldots, \sigma_{k-1}$ are arbitrary real numbers.

Now, the problem of the determination of $\boldsymbol{h}$ consists in solving the system of linear equations (eqn. 3*a*). The 'ill-posedness' of the original problem transforms itself into the possible ill-conditioning of the inversion of the matrix $\boldsymbol{S}$. Notably, due to the arbitrariness in the choice of $\sigma_1, \ldots, \sigma_{k-1}$ there exists an infinite number of matrices $\boldsymbol{S}$ which satisfy eqn. 3*a*. Therefore, we may choose $\sigma_1, \ldots, \sigma_{k-1}$ in such a way that the inversion of the matrix $\boldsymbol{S}$ is as well-conditioned as possible. The set of $\sigma_1, \ldots, \sigma_{k-1}$, which secures a possibly 'best-conditioned' inversion of matrix $\boldsymbol{S}$, will, after Tikhonov, be referred to as the regularisor.[1,2]

Choice of optimum regularisor: There are quite a number of measures of ill-conditioning with respect to matrix inversion.[3] In this work, the determinant of the normalised matrix is used. The matrix $\boldsymbol{S}$ is normalised by dividing the nth row ($n = 1, \ldots, r$) by a_n given as

$$a_n = \left[\sum_{i=1}^{r} a_{n,i}^2\right]^{1/2} \tag{4}$$

where $a_{n,i}$ are the elements of the matrix $\boldsymbol{S}$ from eqn. 3*a*.

If the determinant of the normalised matrix is close to ± 1 or to 0 then the inversion is well-conditioned or ill-conditioned, respectively. Consequently, our task is to choose the regularisor, i.e. the set of $\sigma_1, \ldots, \sigma_{k-1}$, such that the determinant of the normalised matrix $\boldsymbol{S}$ takes the maximum possible value.

From eqn. 3*a* it follows that $\boldsymbol{S}$ is the lower triangular matrix. Hence, one can write

$$|\det \hat{\boldsymbol{S}}| = \frac{|s_1^m|}{\prod_{j=1}^{m} a_j} \prod_{i=1}^{k-1} \hat{\sigma}_i \tag{5}$$

where $\hat{\boldsymbol{S}}$ is the normalised matrix $\boldsymbol{S}$ and

$$\hat{\sigma}_i \equiv \frac{|\sigma_i|}{a_{m+i}} \tag{6}$$

Now, we define

$$\sigma \equiv \prod_{i=1}^{k-1} \hat{\sigma}_i \tag{7}$$

and call it the parameter of regularisation.

For the assumed input signal s the most we can do for obtaining the possibly 'best-conditioned' inversion of $\boldsymbol{S}$ is to choose the value of the parameter of regularisation as close to 1 as possible, i.e. $\sigma \cong 1$. From eqns. 6 and 7 it follows that this requirement can be easily met by choosing appropriately large values of σ_i, $i = 1, \ldots, k-1$, so that each of the $\hat{\sigma}_i$ is also as close to 1 as possible. Notably, the absolute value of the determinant of $\hat{\boldsymbol{S}}$ (eqn. 5) is dependent on both the parameter of regularisation σ and the input signal itself. To establish the influence of the input signal on the conditioning of the inversion of $\boldsymbol{S}$ assume that the value of σ is optimum, i.e. $\sigma \cong 1$. Then, in virtue of eqns. 5 and 7 it follows that

$$\det \hat{\boldsymbol{S}} \cong \frac{s_1}{a_1}\frac{s_1}{a_2}\ldots\frac{s_1}{a_m} \tag{8}$$

To have the value of eqn. 8 as close as possible to ± 1 the ratios $s_1/a_1, s_1/a_2, \ldots, s_1/a_m$ should also be as close as possible to ± 1. One may find that

$$\min\left\{\frac{|s_1|}{a_i}\right\} = \frac{|s_1|}{a_m} = \frac{|s_1|}{\left(\sum_{i=1}^{k} s_i^2\right)^{1/2}} \qquad i = 1, \ldots, m \tag{9}$$

Hence the value of eqn. 9 is the crucial parameter dependent on signal s that determines whether the inversion of $\boldsymbol{S}$ is well-conditioned or not. Therefore, it is suggested here that the value of eqn. 9 be the condition number for the inverse problem discussed here, i.e.

$$\text{cond}\,(s) = \frac{|s_1|}{\left(\sum_{i=1}^{k} s_i^2\right)^{1/2}} \tag{10}$$

Practically, the value of cond (s) can be considered as a measure of the appropriateness of the signal s as a test signal for the nonparametric system identification. If the regularisator is a 'good' one (i.e. close to 1), and cond (s) is also close to 1, i.e. the signal s is also 'good', the inverse of $\boldsymbol{S}$ is always well-posed. On the other hand, if the cond (s) is close to 0, the inverse of S is ill-posed despite the regularisor being a 'good' one. In virtue of eqn. 10, a 'good' signal s should have most of its energy concentrated in its first sample s_1. The best possible signal in this sense is a single pulse for which cond $(s) = 1$. In this case, however, no inversion is needed as the output signal g equals the impulse response h that is sought.

Illustrative example: To examine the influence of both 'good' and 'bad' regularisor and signal s on the accuracy of the inversion of S, consider the following example of the system input signal and its impulse response to be identified, respectively: $s = \{s_1, s_2\}$, $h = \{h_1, h_2\}$. Suppose that the true output signal $g = \{g_1, g_2, g_3\}$ is determined with errors ε_1, ε_2 and ε_3, respectively. Using routine transformations one can find that the inaccurate determination of g affects the estimation of true values of h in two ways:

(*a*) It causes the spurious sample h_3 to appear in the estimated impulse response. The magnitude of this sample depends on both the cond (s) and the parameter of regularisation σ.

(*b*) It causes the inaccuracy in the determination of the true samples h_1 and h_2. The degree of inaccuracy depends on cond (s) only.

The same conclusions are true for s, h and g being longer sequences. In this case the number of spurious samples that appears in the estimated impulse response is $k - 1$.

To illustrate the behaviour of the error in the determination of the impulse response h, four combinations of a 'good' or 'bad' signal s and the regularisator are taken into account and the results of computations are presented in Table 1. These results are self-explanatory and require no further comment.

Table 1 ERROR BEHAVIOUR

Impulse response identified
True value $h_1 = 3$, $k_2 = 1$, $h_3 = 0$

Errors in determination of output signal g
$\varepsilon_1 = 0{\cdot}1$, $\varepsilon_2 = -0{\cdot}1$, $\varepsilon_3 = 0{\cdot}1$

	'good' input signal $s = \{10, 1\}$ cond $(s) \simeq 0{\cdot}995$	'bad' input signal $s = \{1, 10\}$ cond $(s) \simeq 0{\cdot}0995$
'good' regularisor $\sigma \simeq 0{\cdot}995$	$h_1 \simeq 3{\cdot}01$, $h_2 \simeq 0{\cdot}989$ $h_9 \simeq 0{\cdot}0111$	$h_1 \simeq 3{\cdot}1$, $h_2 \simeq -0{\cdot}1$ $h_3 \simeq 0{\cdot}111$
'bad' regularisor $\sigma \simeq 0{\cdot}0995$	$h_1 \simeq 3{\cdot}01$, $h_2 \simeq 0{\cdot}989$ $h_3 \simeq 1{\cdot}11$	$h_1 \simeq 3{\cdot}1$, $h_2 \simeq -0{\cdot}1$ $h_3 \simeq 11{\cdot}1$

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FREQUENCY-DEPENDENT ANALYSIS OF FINLINE MIXED-TYPE OFFSET JUNCTIONS

Indexing terms: Microwave devices and components, Frequency-domain analysis

The frequency-dependent characteristics of finline mixed-type offset junctions are analysed by employing a mode-matching technique in conjunction with a spectral-domain method. Some numerical results are presented.

Introduction: Finlines have been widely applied in millimetre-wave integrated circuits. The general finline step junction as shown in Fig. 1 is the most frequently encountered structure in the circuit design. Therefore, it is important to develop analytical techniques to characterise the step junction. However, up to this date most of the theoretical work has been concerned with the analysis of coaxial ($\Delta = 0$) or offset ($\Delta \leq |w_1 - w_2|/2$) step junctions;[1-4] only a relatively small effort has been made to analyse mixed-type offset ($\Delta > |w_1 - w_2|/2$) step junctions,[5] and their frequency-dependent behaviour has not been reported.

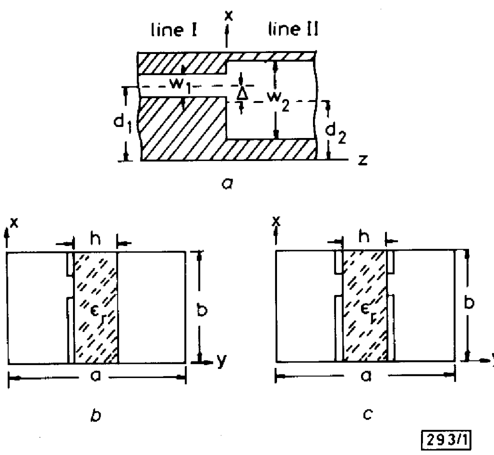

Fig. 1 *General finline step junction*
a Longitudinal-section
b Cross-section of unilateral finline
c Cross-section of bilateral finline

This letter presents a rigorous and efficient hybrid-mode approach for the analysis and characterisation of finline mixed-type offset junctions. The approach is based on the mode-matching technique and the spectral-domain method. In the result the formulation of the generalised scattering matrix which represents the propagating as well as the evanescent hybrid-mode scattering properties of this kind of finline discontinuity is arrived at. The frequency-dependent behaviour of a unilateral finline mixed-type offset junction is illustrated.

Theoretical approach: As a matter of fact, the major complexity of the analysis of a finline mixed-type offset junction lies in a mixed-type boundary which consists of two conducting obstacle surfaces S_I and S_{II} for lines I and II, respectively, beside a common aperture surface S_A. For the treatment of this kind of boundary problem, an auxiliary finline section (line III) is introduced, as shown in Fig. 2. Next, the EM fields in each homogeneous line are expanded in terms of the hybrid modes HE_m and EH_m as follows:

$$\begin{aligned} E^{(v)} &= \sum_m \nabla \times \nabla \times \Pi_{em}^{(v)} - j\omega\mu\nabla \times \Pi_{hm}^{(v)} \\ H^{(v)} &= \sum_m j\omega\varepsilon\nabla \times \Pi_{em}^{(v)} + \nabla \times \nabla \times \Pi_{hm}^{(v)} \quad v = \text{I, II, III} \end{aligned} \tag{1}$$

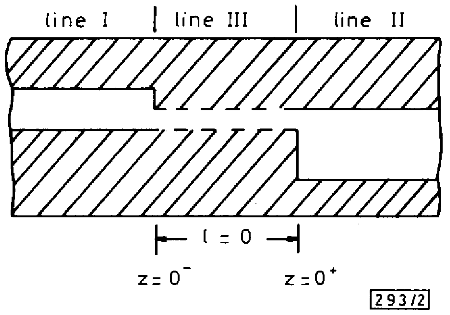

Fig. 2 *Equivalent model for finline mixed-type offset junction*