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A NONITERATIVE ALGORITHM FOR DECONVOLUTION-INVERSE FILTERING USING THE CHEBYSHEV MINIMAX NORM FOR THE APPROXIMATION ERROR

Part I: Theory

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ABSTRACT: A new computational noniterative algorithm which gives the solution to a linear deconvolution-inverse filtering problem is proposed and its properties are studied. It is proved, in some specific cases of input signal, that the algorithm discussed gives the solution, which is equal to that with the Chebyshev minimax norm for the approximation error. In a general case of input signal the solution obtained provides a good prompt for determining an "appropriate subsystem" of $n + 1$ linear equations of n unknowns, which directly gives the Chebyshev minimax norm based solution.

INTRODUCTION

A problem of considerable importance in numerous areas of science and technology is the deconvolution-inverse filtering of an assumed input signal. Usually, an ideal deconvolution-inverse filter is defined as a circuit or a computational algorithm which, for a defined input signal, responds with the Dirac's delta or Kronecker's delta, in the case of continuous or sampled signals, respectively. Unfortunately, deconvolution-inverse filtering defined in such a way is not feasible, and therefore one has to accept the approximate solutions. The approximation error manifests itself through the appearance of additional undesirable pulses in the output signal, which are referred to as the sidelobes.

For practical reasons the problem of deconvolution-inverse filtering has usually been attacked in the discrete time domain. Current approaches to this problem include Wiener filtering e.g., [1], Truncated-Inverse-Filter method [2], Zero-Forcing-Filter method [3], and many others. The common disadvantage of most of them is, that they do not permit to control the output signal sidelobe amplitudes, relative to the mainlobe amplitude. This disadvantage can be avoided by using the Chebyshev minimax norm for the approximation error. Consequently this leads to the Chebyshev solution of inconsistent linear equations system. Unfortunately, the latter can only be obtained using lengthy iterative procedures [4], which for most engineering applications might be hard to accept.

The aim of this paper is to introduce and examine some properties of a new computational noniterative algorithm, referred to hereafter as the D-algorithm, presented briefly in [5]. This is for a general case of input signal and gives a solution to the linear deconvolution-inverse filtering problem, being close to the solution with the Chebyshev minimax norm for the approximation error. The latter from now on will be referred to as the Chebyshev minimax solution or the CMS. We shall also introduce and discuss the necessary and sufficient conditions which ensure that the D-algorithm solution equals the CMS.

The organization of the paper is as follows. Section 1 formulates the minimax problem, Section 2 explains the linearisation method and briefly recalls the de La Vallee Poussin method of computing the Chebyshev solution of inconsistent linear equations system, Section 3 introduces the D-algorithm. Section 4 formulates the criterion (K) for the determination of an "appropriate system" of functions, and briefly examines the algorithm properties. More detailed discussion on the algorithm performance has been presented in the second part of this paper subtitled "Performance" [6].

1. FORMULATION OF THE MINIMAX PROBLEM

In this work, apart from its general scope, we shall focus attention on determining the possible input signals for which the D-algorithm proposed in [5] (and in more detail in Section 3) may give the CMS. Basic considerations presented both, in Section 4 and in the introduction to the second part of this paper [6] which take into account the properties of the Chebyshev solution for inconsistent linear systems of equations, lead to the conclusion that, particularly in the case of input signals being even functions, the D-algorithm discussed may give the CMS. For this reason we shall restrict our considerations to even input signals. It should be noted, that despite this restriction, the solution sought is still of practical importance, since in numerous applications e.g., post matched filter signal processing, reduction of sidelobes in antenna beam-patterns etc., this restriction is met.

Now, consider the following functions,

$$s(n) = \delta(n) + \sum_{i=1}^{q} a_i \circ [\delta(n-i) + \delta(n+i)] , \tag{1.1}$$

$$h_n(x) = \delta(n) + \sum_{i=1}^{p} x_i \circ [\delta(n-i) + \delta(n+i)] , \tag{1.2}$$

$$g_n(x) = s(n) * h_n(x) = \sum_{r} s(r) \circ h_{n-r}(x) , \tag{1.3}$$

where q, p are the fixed natural numbers, $a_1, \ldots, a_q$ are fixed real numbers, $x = (x_1, \ldots, x_p)$ are the variables, and n is an integer,

$$\delta(n) \equiv \begin{Bmatrix} 1, & n = 0 \\ 0, & n \neq 0 \end{Bmatrix} ,$$

and the asterisk denotes convolution.

In terms of linear system theory, $s(n)$ stands for the input signal, $h_n(x)$ is the impulse response sought, and $h_n(x)$ is the output signal.

The impulse response $h_n(x)$ is to be derived subject to the following minimax criterion,

$$\inf_x \max_{n\neq 0}[|g_n(x)|\circ|g_0(x)|^{-1}], \tag{1.4}$$

where $g_0(x)$ and $g_n(x)$, $n\neq 0$, are referred to as the output signal mainlobe and the sidelobes, respectively. Consequently, this criterion can be formulated as the following minimax problem (M).

(M). Determine the system $\hat{x}=(\hat{x}_1,\ldots,\hat{x}_p)$, for which (1.4) is attained.

It has been proved in [7], that if the infimum (1.4) is to be attained, then the fact that the input signal is an even function, implies that the impulse response $h_n(x)$ and consequently the output signal $g_n(x)$ must both be even functions too.

2. SOLUTION OF THE MINIMAX PROBLEM (M) USING THE METHOD OF VARIABLE LINEARISATION

Direct application of the criterion (1.4) leads to a nonlinear minimax problem (M). In order to have it transformed into a linear one we proceed as follows. From eq. (1.1) to eq. (1.3) we find that $g_n(x)$ are the polynomials of the first order. In particular we have,

$$g_0(x)=1+2\circ\sum_{i=1}^{k} a_i\circ x_i\,,\quad k=\min\{q,p\}\,. \tag{2.1}$$

Consider the new variables,

$$t_i=x_i\circ\left(1+2\circ\sum_{j=1}^{k} a_j\circ x_j\right)^{-1},\quad i=1,\ldots,p\,. \tag{2.2}$$

Having multiplied eq. (2.2) by $2\circ a_i$ and then summed it up from 1 to k over i, we have the following,

$$\left(1+2\circ\sum_{j=1}^{k} a_j\circ x_j\right)^{-1}=1-2\circ\sum_{j=1}^{k} a_j\circ t_j\,. \tag{2.3}$$

From eq. (2.2) and eq. (2.3) it follows that,

$$x_i=t_i\circ\left(1-2\circ\sum_{j=1}^{k} a_j\circ t_j\right)^{-1},\quad i=1,\ldots,p\,. \tag{2.4}$$

Having designated,

$$N_0=\{1,\ldots,p+q\}\,, \tag{2.5}$$

and making use of eqs. (2.1), (2.2), and eq. (2.3) we have,

$$r_n(t)\equiv\frac{g_n(x)}{g_0(x)}=\sum_{i=1}^{p} a_{n,i}\circ t_i-b_n\,,\quad n\in N_0\,, \tag{2.6}$$

where $t=(t_1,\ldots,t_p)$ and $a_{n,i}$, b_n are certain real numbers. Thus, the nonlinear minimax problem (M) has been transformed into the following linear one, designated as $(\hat{M}_0)$.

$(\hat{M}_0)$. Determine the system $\hat{t} = (\hat{t}_1, \ldots, \hat{t}_p)$ for which

$$\inf_t \max_{n \in N_0} |r_n(t)| \tag{2.7}$$

is attained.

The above problem can be solved by using one of the iterative methods described in [4], provided that the Haar condition is satisfied. With reference to eq. (2.6) the Haar condition means, that every system of p vectors chosen out of the $A_1, \ldots, A_{q+p}$ vectors, where $A_i = (a_{i,1}, \ldots, a_{i,p})$, should be linearly independent. This condition is equivalent to the requirement that every minor of the rank p of the following matrix

$$[A] = \begin{bmatrix} a_{1,1}, \ldots\ldots, a_{1,p} \\ \ldots\ldots\ldots\ldots \\ a_{p+q,1}, \ldots\ldots, a_{p+q,p} \end{bmatrix} \tag{2.8}$$

be nonzero.

Now, on the basis of [4], (Sections 3.3 and 3.4), we shall briefly present the way in which the solution to the problem $(\hat{M}_0)$ can be derived.

First, let

$$N = \{n_1, \ldots, n_{p+1}\}, \quad n_1 < \cdots < n_{p+1}, \tag{2.9}$$

be a subsystem of system N_0 given by eq. (2.5).

Now, consider the following subsystem,

$$r_n(t), \quad n \in N, \tag{2.10}$$

of the system (2.6). For system (2.10) the following reduced minimax problem $(\hat{M})$ can be formulated.

$(\hat{M})$. Determine the system $\hat{t} = (\hat{t}_1, \ldots, \hat{t}_p)$ for which

$$\inf_t \max_{n \in N} |r_n(t)| \tag{2.11}$$

is attained.

To find the solution to $(\hat{M}_0)$ and $(\hat{M})$ we shall use the following theorem [4].

THEOREM: Every solution of the problem $(\hat{M}_0)$ is a solution of the problem $(\hat{M})$ for an appropriate subsystem (2.10).

Once such an "appropriate subsystem" is found the minimax solution sought can be obtained in the following manner. Suppose, that by some means we know the "appropriate subsystem" (2.10) specified by the "appropriate subsystem of indices" N, (2.9). Then, we proceed using the method of de La Vallee Poussin, as follows [4].

Consider the following system of homogeneous equations,

$$\sum_{n \in N} \lambda_n \circ a_{n,i} = 0, \quad i = 1, \ldots, p+1, \tag{2.12}$$

with the unknowns λ_n, $n \in N$, (2.9).

By the Haar condition assumed, the system (2.12) has a nonzero solution, i.e., $\lambda_n \neq 0$, for $n \in N$.

Now, designate,

$$h = -\Big(\sum_{n\in N} \lambda_n \circ b_n\Big) \circ \Big(\sum_{n\in N} |\lambda_n|\Big)^{-1}, \tag{2.13}$$

and consider the following system of $p+1$ equations

$$r_n(t) = h \circ \operatorname{sign} \lambda_n, \quad n \in N, \tag{2.14}$$

with the variables $t_1, \ldots, t_p$.

Having solved any subsystem of system (2.14), consisting of p equations we obtain the following unique solution to system (2.14),

$$\hat{t} = (\hat{t}_1, \ldots, \hat{t}_p). \tag{2.15}$$

The number $|h|$ given by eq. (2.13) is the common value of the expressions (2.7) and (2.11) while system (2.15) is the unique solution to both $(\hat{M}_0)$ and $(\hat{M})$. Once the solution (2.15) is obtained, one has to return via eq. (2.4) to the original variables x_i, obtaining thus the solution to our original minimax problem (M).

By eq. (2.4), it follows that $\hat{x} = (\hat{x}_1, \ldots, \hat{x}_p)$, with

$$\hat{x}_i = \hat{t}_i \circ \Big(1 - 2 \circ \sum_{j=1}^{p} a_j \circ \hat{t}_j\Big)^{-1}, \quad i = 1, \ldots, p, \tag{2.16}$$

is the unique solution to the minimax problem (M) and $|h|$ is the value of the infimum (1.4). It is noted, that the following condition is to be met,

$$2 \circ \sum_{j=1}^{p} a_j \circ \hat{t}_j \neq 1. \tag{2.17}$$

In addition, if the system (2.10) is to be the "appropriate subsystem" of system (2.6), as assumed, then the following inequality must be satisfied,

$$|r_n(\hat{t})| \leq |h|, \quad n \in N_0 \backslash N. \tag{2.18}$$

If (2.18) is not met, then system (2.10) is not the "appropriate subsystem" assumed.

Unfortunately, as a rule the "appropriate subsystem" (2.10) is unknown and the main expenditure of effort is in finding this subsystem, particularly for large values of q. Several algorithms, e.g. ascent algorithm, descent algorithm, etc., enabling an efficient search for the "appropriate subsystem" are known [4]. However, all of them require a certain number of iterations, which depend on the closeness of our initial guessing of the "appropriate subsystem" to the solution of the problem $(\hat{M}_0)$.

3. NONITERATIVE D-ALGORITHM OF APPROXIMATE SOLUTION TO THE MINIMAX PROBLEM (*M*)

As already mentioned, the algorithm discussed here has been proposed and briefly described in [5], showing it as a parallel combination of the finite-length Zero-

Forcing-Filters [3]. We shall present here another more mathematically rigorous description of this algorithm which is necessary for further analysis.

The main assumption and at the same time restriction on this algorithm, is, with reference to eqs. (1.1), (1.2), and (1.3), the following

$$q = k\,, \quad p = k \circ l\,, \tag{3.1}$$

where k and l are positive integers.

Now designate,

$$x^j = (x_1^j, \ldots, x_{jk}^j)\,, \quad j = 1, \ldots, l\,, \tag{3.2}$$

$$h_n^j(x^j) = \delta(n) + \sum_{i=1}^{j \circ k} x_i^j \circ [\delta(n-i) + \delta(n+i)]\,, \tag{3.3}$$

$$g_n^j(x^j) = s(n) * h_n^j(x^j)\,. \tag{3.4}$$

Consider the following system of equations,

$$g_n^1(x^1) = 0\,, \quad n = 1, \ldots, k\,. \tag{3.5}$$

Let $\hat{x}^1 = (\hat{x}_1^1, \ldots, \hat{x}_k^1)$ be a solution to system (3.5). Designate,

$$\left.\begin{aligned} &\varepsilon_1 = \max\{|g_n^1(\hat{x}^1)|: \quad n = k+1, \ldots, 2k\}\,, \\ &a = \max|a_n|,\ z_1 = \frac{\varepsilon_1}{a} \circ \operatorname{sgn} g_0^1(\hat{x}^1)\,, \\ &\hat{g}_n(z_1) = z_1 \circ s(n)\,. \end{aligned}\right\} \tag{3.6}$$

Consequently, consider the system of equations,

$$g_n^2(x^2) = 0\,, \quad n = 1, \ldots, 2k\,. \tag{3.7}$$

Let $\hat{x}^2 = (\hat{x}_1^2, \ldots, \hat{x}_{2k}^2)$ be a solution to system (3.7). Designate,

$$\left.\begin{aligned} &\varepsilon_2 = \max\{|g_n^2(\hat{x}^2)|: \quad n = 2k+1, \ldots, 3k\}\,, \\ &z_2 = \frac{\varepsilon_1}{\varepsilon_2} \circ [\operatorname{sgn} g_0^2(\hat{x}^2)] \circ [\operatorname{sgn} g_0^1(\hat{x}^1)]\,. \end{aligned}\right\} \tag{3.8}$$

Eventually, consider the following system of equations,

$$g_n^l(x^l) = 0\,, \quad n = 1, \ldots, lk\,. \tag{3.9}$$

Let $\hat{x}^l = (\hat{x}_1^l, \ldots, \hat{x}_{lk}^l)$ be a solution to system (3.9). Designate,

$$\left.\begin{aligned} &\varepsilon_l = \max\{|g_n^l(\hat{x}^l)|: \quad n = lk+1, \ldots, (l+1) \circ k\}\,, \\ &z_l = \frac{\varepsilon_1}{\varepsilon_l} \circ [\operatorname{sgn} g_0^l(\hat{x}^l)] \circ [\operatorname{sgn} g_0^1(\hat{x}^1)]\,. \end{aligned}\right\} \tag{3.10}$$

Now, consider the functions,

$$\hat{g}^l_n(x^1, \ldots, x^l) = \hat{g}_n(z_1) + g^1_n(x^1) + z_2 \circ g^2_n(x^2) + , \ldots , z_l \circ g^l_n(x^l) . \tag{3.11}$$

From eq. (3.5) to eq. (3.11) it follows that,

$$\left.\begin{aligned}
&\hat{g}^l_n(\hat{x}^1, \ldots, \hat{x}^l) = \hat{g}_n(z_1) = a_n \circ z_1 , && n = 1, \ldots, k ,\\
&\hat{g}^l_n(\hat{x}^1, \ldots, \hat{x}^l) = g^1_n(\hat{x}^1) , && n = k+1, \ldots, 2k ,\\
&\hat{g}^l_n(\hat{x}^1, \ldots, \hat{x}^l) = z_2 \circ g^2_n(\hat{x}^2) , && n = 2k+1, \ldots, 3k ,\\
&\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots && \\
&\hat{g}^l_n(\hat{x}_1, \ldots, \hat{x}_l) = z_l \circ g^l_n(\hat{x}^l) , && n = lk+1, \ldots (l+1) \circ k .
\end{aligned}\right\} \tag{3.12}$$

By eq. (3.4) and eq. (3.6) one finds that functions (3.11) can be interpreted as the output signal of the filter representing the D-algorithm and can be expressed as:

$$\hat{g}^l_n(x^1, \ldots, x^l) = s(n) * [z_1 \circ \delta(n) + h^1_n(x^1) + z_2 \circ h^2_n(x^2) + \cdots + z_l \circ h^l_n(x^l)] \tag{3.13}$$

In terms of linear filtering theory eqs. (3.5), (3.7) and (3.9) define the Zero-Forcing-Filters (ZFFs) of the 1st, 2nd and 1th order, while $h^1_n(\hat{x}^1)$, $h^2_n(\hat{x}^2)$, and $h^l_n(\hat{x}^l)$ represent their impulse responses, respectively. The $\delta(n)$ in eq. (3.13) can be interpreted as the impulse response of the zero order ZFF, while $z_1, z_2, \ldots, z_l$ are the weighting coefficients. These cause the maximum sidelobe amplitudes at the output of all ZFFs to be equal.

Now, designate,

$$x_n = \frac{h^1_n(x^1) + z_2 \circ h^2_n(x^2) + \cdots + z_l \circ h^l_n(x^l)}{1 + z_1 + \cdots + z_l} , \tag{3.14}$$

$$n = 1, \ldots, kl , \quad x = (x_1, \ldots, x_{kl}) .$$

Then, using eqs. (1.2), (3.1) and (3.3) we have,

$$z_1 \delta(n) + h^1_n(x^1) + z_2 h^2_n(x^2) + \cdots + z_l h^l_n(x^l) = (1 + z_1 + \cdots + z_l) \circ h_n(x) .$$

Hence, by eqs. (3.13), (1.3) and (3.1) one obtains,

$$\hat{g}^l_n(x^1, \ldots, x^l) = (1 + z_1 + \cdots + z_l) \circ g_n(x) . \tag{3.15}$$

Now, with reference to eq. (3.14) we also designate,

$$\hat{x}_n = \frac{h^1_n(\hat{x}^1) + z_2 \circ h^2_n(\hat{x}^2) + \cdots + z_l \circ h^l_n(\hat{x}^l)}{1 + z_1 + \cdots + z_l} , \quad n = 1, \ldots, kl, \quad \hat{x} = (\hat{x}_1, \ldots, \hat{x}_{kl}) , \tag{3.16}$$

and

$$H \equiv \varepsilon_1 \circ |\hat{g}_0^l(\hat{x}^1, \ldots \hat{x}^l)|^{-1} = \varepsilon_1 \circ [(1 + z_1 + \cdots + z_l) \circ |g_0(\hat{x})|]^{-1} . \tag{3.17}$$

Then, eqs. (3.6), (3.8), (3.10), (3.12), (3.15), and (3.16) imply that,

$$\max_{n \neq 0} [|g_n(\hat{x})| \circ |g_0(\hat{x})|^{-1}] = H . \tag{3.18}$$

From eq. (3.18) it follows that,

$$\inf_x \max_{n \neq 0} [|g_n(x)| \circ |g_0(x)|^{-1}] \leq H . \tag{3.19}$$

Hence, it follows that the number H is a certain, generally overestimated value of the infimum (1.4), and consequently, $\hat{x} = (\hat{x}_1, \ldots, \hat{x}_{kl})$ is the corresponding approximate solution to the minimax problem (M).

4. CRITERION (K) FOR CHOICE OF THE "APPROPRIATE SYSTEM"

The aim of this section is to formulate a criterion, which on the basis of the approximate solution obtained with the D-algorithm, would help us to select the "appropriate subsystem" of the functions $r_n(t)$, (2.10) of the system (2.6). Consequently, the "appropriate subsystem of indices" N (2.9), can be chosen using the following criterion, hereafter referred to as criterion (K). Prior to the formulation of this criterion we should note that due to the assumptions (3.1) the system N_0, defined by eq. (2.5) takes the following form,

$$N_0 = \{1, \ldots, (l + 1) \circ k\} \tag{4.1}$$

CRITERION (K). It is given the system (3.12). As the "appropriate subsystem of indices" N of the system N_0, we choose the system consisting of $kl + 1$ elements of N_0 such, that each of the numbers,

$$|\hat{g}_n^l(\hat{x}^1, \ldots, \hat{x}^l)| , \quad n \in N , \tag{4.2}$$

is larger than any of the following numbers,

$$|\hat{g}_n^l(\hat{x}^1, \ldots, \hat{x}^l)| , \quad n \in N_0 \backslash N . \tag{4.3}$$

In other words, Criterion K states that, after having used the D-algorithm, we should choose those $kl + 1$ indices, for which the $kl + 1$ largest output signal sidelobes appear.

Now, taking into account eq. (3.15) we find, that $|\hat{g}_n^l(\hat{x}^1, \ldots, \hat{x}^l)|$ can be repalced by $|g_n(\hat{x})|$, when using criterion (K). So, each of the numbers $|g_n(\hat{x})|$, $n \in N$, is larger than any of the numbers $|g_n(\hat{x})|$, $n \in N_0 \backslash N$. Hence, by eq. (3.18) we have,

$$H = \max_{n \in N} [|g_n(\hat{x})| \circ |g_0(\hat{x})|^{-1}] . \tag{4.4}$$

Moreover, the condition,

(C_1) – all numbers $|g_n(\hat{x})|$, $n \in N$, are equal is equivalent to
(C_2) – all numbers (4.2) are equal.
Hence, if (C_2) is not met, then (C_1) is not met either, which consequently, by eqs. (3.18), (3.19), and (4.4) implies the following inequality,

$$\inf_x \max_{n \neq 0} [|g_n(x)| \circ |g_0(x)|^{-1}] < H , \tag{4.5}$$

This inequality and eq. (3.18) show, that the system (3.16) is the approximate soluton to (M) only. If (C_2) is met, then the system (3.16) may be, (but not necessarily), the CMS to the problem (M). To discuss both cases in more detail we proceed as follows. The linearisation method described in Section 2 is used to transform the nonlinear problem (M) into the linear problem $(\hat{M}_0)$. Then, using criterion (K) we reduced the $p + q$ minimax problem $(\hat{M}_0)$ into a $p + 1$ minimax problem $(\hat{M})$. Consequently, using the de La Vallee Poussin method we find the solution $\hat{t}$, (2.15) and the value of h, (2.13). Now, two cases are possible:

$$\begin{aligned} &1. \quad |h| = H \quad \text{and} \\ &2. \quad |h| < H . \end{aligned} \tag{4.6}$$

Ad. 1. With reference to eq. (3.19) and in addition having the following,

$$\begin{aligned} |h| &= \inf_t \max_{n \in N} |r_n(t)| \leq \inf_t \max_{n \in N_0} |r_n(t)| = \\ &= \inf_x \max_{n \neq 0} [|g_n(x)| \circ |g_0(x)|^{-1}] , \end{aligned}$$

we find that,

$$\inf_x \max_{n \neq 0} [|g_n(x)| \circ |g_0(x)|^{-1}] = H . \tag{4.7}$$

Hence, by eq. (3.18) system (3.16) yields the CMS to the problem (M). Moreover, the condition (C_2) is met and the criterion (K) proves to be effective. It should be noted that for even input signals with sidelobes of equal amplitudes and arbitrary signs we can always make condition (C_2) to be met for $l = 1$, in eq. (3.1). Moreover, in the specific case of input signal with two sidelobes criterion (C_2) can be met for any value of l and the solution obtained equals the CMS [8]. Detailed discussion on these properties of the D-algorithm has been presented in the second part of this paper subtitled "Performance" [6].

Ad. 2. In this case, irrespective of (C_2), the inequality (4.5) holds. Hence, as already mentioned, system (3.16) yields an approximate solution to the problem (M). To verify the criterion (K) we derive the solution (2.15) to system (2.14) for $p = kl$. If the conditions (2.17) and (2.18) are met, then $\hat{t}$, (2.15), is the unique solution to $(\hat{M})$ and $(\hat{M}_0)$, and $|h|$ is the common value of the expressions (2.7) and (2.11). Consequently, system (2.16), for $p = kl$, is the unique solution to (M), while $|h|$ is the value of the infimum (1.4). In addition, this means that the criterion (K) proves to be effective. If in eq. (2.18) is not satisfied, then the criterion (K) fails and the solution is to be derived either by modifying this criterion, or by using one of the iterative procedures described by [4].

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