A R C H I W U M E L E K T R O T E C H N I K I

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Analysis of an active resistance transformer

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An analysis of the d.c. and low frequency upward and downward resistance transformer is presented. Relations between the resistance transformation error and the parameters of operational amplifiers have been derived and experimentally verified. A method of resistance transformation error estimation is proposed. Two modifications of the extending application of the basic circuit are presented.

1. INTRODUCTION

Resistance transformers are used in measurement and control systems when it is necessary to transform a given value of resistance R_x to the desired value of R_i. Such a transformation is given by the equation (1)

$$R_i = nR_x + R_f, \tag{1}$$

where R_i, R_x, R_f are quoted in units of resistance and the transformation ratio n is positive and constant over the range of R_x variation.

Few papers [1, 2] deal with a practical implementation of the equation (1) and describe circuits realizing downward transformation only.

This paper presents analysis of the resistance transformer performing linear upward and downward transformation of resistance.

2. PRINCIPLE OF OPERATION

It is known that a circuit with voltage controlled voltage sources, with transmittance $K(s)$ and with impedance $Z_f(s)$ in the feedback loop (Fig. 1a) has an input impedance $Z_i(s)$ given by (2)

$$Z_i(s) = \frac{Z_f(s)}{1-K(s)}. \tag{2}$$

Depending upon the choice of $K(s)$ and $Z_f(s)$ this circuit leads to different configurations such as: impedance divider [3], capacitance transformer [3], simulated inductance [4] and so on.

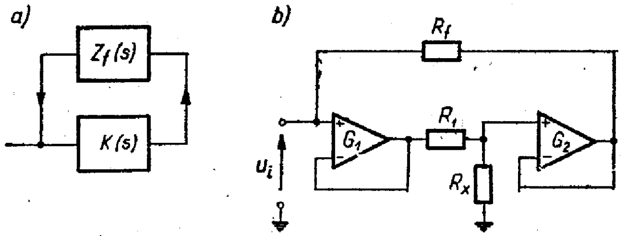

Fig. 1. Active resistance transformer

In the analysed circuit (Fig. 1b) operational amplifiers G_1, G_2 and resistors R_1, R_x form a voltage controlled voltage source with transmittance $K(s)$

$$K(s)=\frac{R_x}{R_1+R_x} \tag{3}$$

and R_f represents a feedback loop

$$Z_f(s)=R_f. \tag{4}$$

From (2), (3) and (4) one obtains

$$Z_i(s)=R_i=\frac{R_f}{R_1}R_x+R_f. \tag{5}$$

The value of resistance R_i can be linearly controlled by varying R_f or R_x. The first method of control by changing R_f is not very useful for two reasons:

- only since $R_x/R_1>0$ it performs upward transformations,
- R_f is „floating" and it might be difficult to control its value.

The second method of control by varying R_x performs both downward and upward transformation. It can be easily realized by varying the value of the grounded resistance R_x. In this case the transformation ratio n can be calculated from (1) and (5) as

$$n=\frac{R_f}{R_1}. \tag{6}$$

3. DERIVATION OF THE TRANSFORMATION ERROR

In practice there is an uncertainty in the R_i transformation. It is caused by the imperfect behaviour of operational amplifiers and the finite tolerances of resistors. To analyse this uncertainly it is first of all assumed that the values of the resistors are perfectly accurate and that errors are introduced by operational amplifiers only. Under this assumption the relative transformation error δ is derived thus:

$$\delta=\left|\frac{R_i-R_i'}{R_i'}\right|, \tag{7}$$

where R_i is the ideal resistance of the transformer and R_i' is the resistance with regard to the sources of error.

Then the tolerances of the resistors and of the measurement set are accounted for and added to δ as the r.m.s. error.

After preliminary examination of the circuit the following sources of error were assumed (fig. 2): input bias current i_b of the operational amplifier, temperature drift $\Delta e_0(\Delta T)$ of the input offset voltage and finite value of *CMRR*. The resultant error was assumed to be equal to the sum of the absolute values of errors introduced by particular sources.

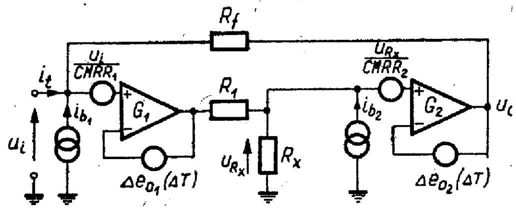

Fig. 2. Equivalent model of the transformer with sources of errors

The circuit in Fig. 2 can be described by the following equations:

$$
\begin{aligned}
i_i &= i_{b_1} + \frac{u_i - u_0}{R_f}, \\
u_0 &= \Delta e_{0_2}(\Delta T) + \left(1 + \frac{1}{CMRR_2}\right) u_{R_x}, \\
u_{R_x} &= \frac{R_x}{R_1 + R_x}\left[u_i\left(1 + \frac{1}{CMRR_1}\right) + \Delta e_{0_1}(\Delta T)\right] - i_{b_2}\frac{R_x R_1}{R_x + R_1}.
\end{aligned}
\tag{8}
$$

Since *CMRR* and $\Delta e_0(\Delta T)$ can be either negative or positive it is necessary to follow the principle of the „worst case" and to assume that the resultant error is the sum of the absolute values of errors introduced by particular sources. Under the assumption that the $CMRR \gg 1$ the relative transformation error δ after routine transformations (appendix I-A3) can be expressed as (9)

$$
\delta = \left[R_x i_{b_2} + R_i i_{b_1} + \left(1 + \frac{R_x}{R_1}\right)\Delta e_{0_2}(\Delta T) + \frac{R_x}{R_1}\Delta e_{0_1}(\Delta T)\right]\frac{1}{|u_i|} + \frac{R_x}{R_1}\left(\frac{1}{CMRR_1} + \frac{1}{CMRR_2}\right).
\tag{9}
$$

In the case of identical operational amplifiers equation (9) can have the form (10), (Appendix I-A4)

$$
\delta = \left[(R_x + R_i)\, i_b + \left(1 + 2\frac{R_x}{R_1}\right)\Delta e_0(\Delta T)\right]\frac{1}{|u_i|} + \frac{R_x}{R_1}\frac{2}{CMRR}.
\tag{10}
$$

4. GRAPHICAL ESTIMATION OF THE TRANSFORMATION ERROR

For designer's application a useful way to present the transformation error δ is to sketch two asymptotes of the equation (10) on a log-log diagram as functions of u_i. The equations describing these asymptotes are

$$\log\delta=\log\left[(R_x+R_i)\,i_b+\left(1+2\frac{R_x}{R_1}\right)\Delta e_0(\Delta T)-\log u_i\right. \tag{11}$$

and

$$\log\delta=\log 2-\log CMRR. \tag{12}$$

This kind of the visualization of the equation (10) allows quick estimation of the transformation error when the parameters of an operational amplifier are given. The asymptotes are also helpful in choosing a suitable operational amplifier and values of R_f and R_1 when the max. transformation error is imposed.

5. EXPERIMENTAL RESULTS

To verify equations (5) and (10) the transformer circuit shown in Fig. 1b was constructed. Two „off-shelf" operational amplifiers 741 were used. The zeroisation of the input offset voltage was introduced in the way recommended by the manufacturer [5]. Input voltage u_i and input current i_i were measured by means of the four-digit DVMs. Four measurements of R_i against u_i ranging from 0.1 mV to 10 V were carried out. The values of the resistors R_f, R_1, R_x were fixed by means of decade resistors with an accuracy of 0.1% determining n, R_x/R_1, and R_i+R_x as in Table 1.

Table 1

	R_x [Ω]	R_1 [Ω]	R_f [Ω]	n	R_i [kΩ]	R_x+R_i [kΩ]	R_x/R_1
I	989,1	1	100	100	99,01	100	1000
II	2846,8	28,5	285	10	28,75	31,6	100
III	4761,9	476,2	476,2	1	5,24	10	10
IV	833,3	833,3	83,3	0,1	0,166	1	1

The measured values of the relative transformation error δ were plotted (continuous line) together with the asymptotes of error (thick continuous line) and values of δ computed theoretically from equation (10) (broken lines) on the common log-log diagram (Fig. 3a, 3b). The curves calculated from the equation (10) as well as the asymptotes do not account for error introduced by the tolerances of resistors and finite accuracy of the measurement set.

It would have been possible to account for this error in equation (10), however this would have made this equation more complicated. It would have also made it difficult to extract from this equation the influence of the parameters of the operational amplifiers on the transformation error δ, which is one of the most important aims of the presented analysis. For those reasons the absolute value of error caused by finite accuracy of DVMs and the tolerances of resistors R_f, R_1, R_x was calculated as a r.m.s. error and presented in the form of the dotted area in the diagram (Fig. 3a, 3b). The lower limit of the dotted area is the theoretical curve of δ calculated from equation (10) while the upper limit is the same curve but with the

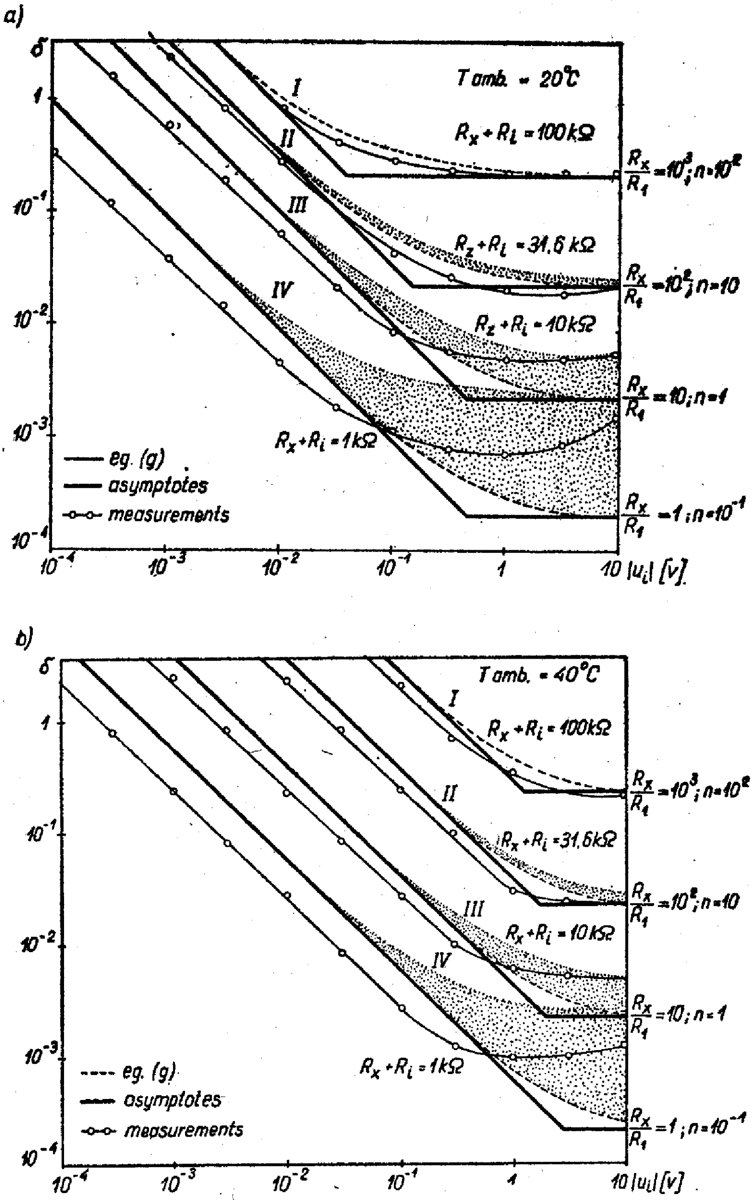

Fig. 3. Relative transformation error δ vs. input voltage $|u_i|$: a) ambient temperature 20°C, b) ambient temperature 40°C

max. value of the r.m.s. error added. Computation of the theoretical value of δ was made for nominal values of parameters of operational amplifiers i.e. $CMRR=80$ dB, $i_b=100$ nA, $\Delta e_0(\Delta T)=\Delta T\cdot 7\cdot 10^{-6}$ V. Two series of measurements were carried out for the same values of R_f, R_1, R_x but for different ambient temperatures: 20 centigrades and 40 centigrades.

6. COMMENT ON THE RESULTS

A comparison of the curves of the relative transformation error δ obtained experimentally and computed from the equation (10) shows good agreement between them and proves the adequacy of the equivalent model of the transformer assumed for the analysis. For small values of u_t transformation error δ is inversely proportional to u_t. For larger values of u_t, where the dominating source of error is the *CMRR*, δ is independent of u_t. This situation is clearly shown of the error asymptotes. For higher values of u_t measured values of δ are higher than the computed ones. In this range the error introduced by the measurement system (finite accuracy of DVMs and tolerances of resistors) is comparable to or higher than the value of δ. The model of the transformer assumed for the analysis is based upon the d.c parameters of the operational amplifiers. This model is also applicable to the a.c analysis provided that the essential parameters of the operational amplifiers in the required frequency range do not differ much from the values of parameters for the d.c. Operational amplifiers 741 exhibit a sharp decrease of value of their *CMRR* and the open loop voltage gain for frequencies above 100 Hz. A test measurement performed by means of a laboratory bridge, with an accuracy of 0.1%, did not show any difference between R_t values measured at the frequencies of 80 Hz and 1 kHz.

7. MODIFICATIONS

As the maximum output current of a typical operational amplifier is limited to 20 mA, it is not possible to obtain very low resistances R_t with reasonable accuracy in case of downward transformation. An improved model of a transformer employing an extra current amplifier allows us to obtain very small values of R_t with the required accuracy – Fig. 4.

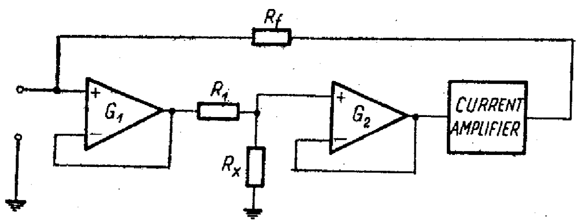

Fig. 4. An improved model of the transformer for very low values of R_t

Another modification consists in the elimination of the operational amplifier G_1. This circuit performs downward transformation only and is equivalent to the structure presented by Parson [1], (Fig. 5). In this case the transformation equation is slightly different from that given by equation (5) and has the form

$$R_t = \frac{1}{n+1}(nR_x + R_f). \tag{13}$$

In this case the relative transformation error is

$$\delta=\frac{1}{n+1}\left\{\left[(n-1)R_x i_b+\left(1+\frac{R_x}{R_1}\right)\Delta e_0(\Delta T)\right]\frac{1}{|u_i|}+\frac{R_x}{R_1}\frac{1}{CMRR}\right\}. \tag{14}$$

This equation can be derived in a similar way as in Appendix I.

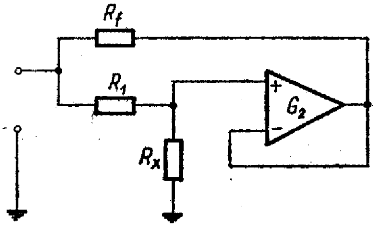

Fig. 5. Simplified circuit for the downward transformation

8. CONCLUSIONS

Comparison of curves of δ versus u_t obtained experimentally and computed from (10) shows good agreement between them both for $\Delta T=0°C$ and $\Delta T=20°C$. For higher values of u_i the measured values of δ differ from computed ones. This is caused by an error introduced by tolerances of resistors and DVMs. In this range of u_i this error is comparable to or higher than the value of δ.

It seems that the transformer presented might be useful in several applications. One particular application for which it was designed, is a realization of the wide range voltage controlled resistor. If R_x is replaced by an FET, voltage controlled r_{ds} of the transistor can be shifted upward or downward by means of this transformer. An examination of the equations (2) and (3) leads to the conclusion that this configuration could be adopted for capacitance and inductance simulation by replacing R_f or R_1 by a capacitor.

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Appendix I

Circuit in Fig. 2 is described by equations (8) hence:

$$i_i=i_{b_1}+\frac{u_i}{R_f}-\frac{\Delta e_{0_2}(\Delta T)}{R_f}-\frac{1}{R_f}\left(1+\frac{1}{CMRR_2}\right)\frac{R_x}{R_1+R_x}\left[u_i\left(1+\frac{1}{CMRR_1}\right)+\Delta e_{0_1}(\Delta T)-i_{b_1}R_1\right]. \tag{A1}$$

Using the definition (7)

$$\delta=\left|\frac{R_i-R_i'}{R_i'}\right|=\left|R_i\frac{i_i}{u_i}-1\right|$$

and introducing i_i from (A1), after routine transformation yields

$$\delta=\left|\left\{R_i i_{b_1}+R_x\left[1+\frac{1}{CMRR_2}\right]-\Delta e_{0_1}(\Delta T)\left[1+\frac{1}{CMRR_2}\right]\frac{R_x}{R_1}-\Delta e_{0_1}(\Delta T)\left[\frac{R_x}{R_1}+1\right]\right\}\frac{1}{u_i}+\right.$$
$$\left.-\frac{R_x}{R_1}\left(\frac{1}{CMRR_1}+\frac{1}{CMRR_2}+\frac{1}{CMRR_1\,CMRR_2}\right)\right|. \quad \text{(A2)}$$

Under the assumption that: $CMRR_1\gg 1$, $CMRR_2\gg 1$

$$\delta=\left[R_i i_{b_1}+R_x i_{b_2}+\frac{R_x}{R_1}\Delta e_{0_1}(\Delta T)+\left(\frac{R_x}{R_1}+1\right)\Delta e_{0_2}(\Delta T)\right]\frac{1}{|u_i|}+\frac{R_x}{R_1}\left(\frac{1}{CMRR_1}+\frac{1}{CMRR_2}\right). \quad \text{(A3)}$$

In the case of two identical operational amplifiers i.e.

$$i_{b_1}=i_{b_2}=i_b,\quad \Delta e_{0_1}(\Delta T)=\Delta e_{0_2}(\Delta T)=\Delta e_0(\Delta T);\quad CMRR_1=CMRR_2=CMRR$$

the relative transformation error can be expressed as

$$\delta=\left\{[R_i+R_x]\,i_b+\Delta e_0(\Delta T)\left[2\,\frac{R_x}{R_1}+1\right]\right\}\frac{1}{|u_i|}+\frac{R_x}{R_1}\,\frac{2}{CMRR}. \quad \text{(A4)}$$

ANALIZA AKTYWNEGO TRANSFORMATORA REZYSTANCJI

W pracy przedstawiono układ transformatora rezystancji umożliwiającego transformację rezystancji zarówno w górę jak i w dół dla składowej stałej oraz zmiennej.

Wyprowadzono i zweryfikowano eksperymentalnie teoretyczne zależności wiążące parametry wzmacniaczy operacyjnych oraz wartości elementów transformatora z błędem transformacji. Podano metodę graficznego oszacowania błędu transformacji oraz przedstawiono dwie modyfikacje układu rozszerzające zakres zastosowań transformatora.

АНАЛИЗ АКТИВНОГО ТРАНСФОРМАТОРА РЕЗИСТАНСА

Представлена схема повышающего и понижающего трансформатора резистанса для постоянной и переменной составляющих.

Выведены и проверены экспериментально-теоретические зависимости ошибки трансформации от параметров операционных усилителей и элементов трансформатора.

Приведен метод графической оценки ошибки трансформации и представлены две модификации схемы расширяющие область применений трансформатора.

DIE ANALYSE DES AKTIVEN RESISTANZ-TRANSFORMATORS

In dieser Arbeit ist der active Resistanz-Transformator vorgestellt, der lineare Resistanz-Transformation nach oben und unten für Gleich- und NF-Wechselspannungsanteil ermöglicht. Es wurden die Zusammenhänge abgeleitet, die den relativen Transformationsfehler mit den Parametern der Operationsverstärker und den dem Transformator zugehörenden Resistorwerten binden. Es ist eine experimentalle Verifikation der theoretischen Ergebnisse durchgeführt worden.

Es wurde eine grafische Methode der Transformationsfehlerabschatzung sowie zwei Modifikationen angegeben, die den Einsatzbereich des Transformators vergrössern.