APPLICATIONS OF CHEBYSHEV MINIMAX DECONVOLUTION FILTERING TO THE ESTIMATION OF DETRENDED DATA*

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Signal derivative at the output of Chebyshev deconvolution filter with respect to the odd pair of impulses is an estimator, very close to the detrended input signal. Computations using a significantly large database of market quotations show that average "closeness" in terms of normalized in L_2 covariance coefficient is above 96 percent.

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1. Introduction

Deconvolution filtering plays an important role in such problems when one wants to invert effect of linear filtering process described by the operation of convolution. As a rule, deconvolution represents a class of ill-posed problems for which solutions are usually unstable or numerically ill-conditioned *e.g.* [1, 2]. In this contribution application of a FIR, (Finite Impulse Response), deconvolution filter with respect to the odd pair of impulses, for processing market data, is discussed. The optimum Chebyshev minimax norm solution for this case of deconvolution filtering has been first presented by Dyka in [3]. It was found that the intrinsic property of the said Chebyshev minimax deconvolution filter is its ability to extract the varying part of the input signal. More specifically, the first order difference (numerical derivative) at the output of the filter is very close, (*i.e.* strongly correlated), to the detrended value of input signal. This correlation in terms of normalized in L_2 covariance coefficient is above 96 percent. In financial time series analysis this property of Chebyshev minimax deconvolution filter can, among others be used for techniques of determining Hurst exponent, which require extraction of the detrended value of input signal, *i.e.* [4].

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2. Deconvolution filter

Impulse response of Chebyshev minimax deconvolution filter with respect to odd pair of impulses takes the following form, [3]:

TABLE I

Chebyshev deconvolution filter	
n	impulse response
1	$(-1,1)$
2	$(-1,-2,2,1)$
3	$(-1,-2,-3,3,2,1)$
4	$(-1,-2,-3,-4,4,3,2,1)$
...	....
...	and so forth

In principle the said deconvolution filter should be considered as a specific integrator *i.e.*, averaging filter. The mean value of impulse response of a typical averaging filter is a positive number, which results in integrating the mean value of input signal. However, the mean value of the Chebyshev deconvolution filter discussed here equals zero, which results in integrating the varying part of the input signal only. This is a very interesting intrinsic property of the Chebyshev deconvolution filter. It enables us to extract from the input signal the varying component by suppressing the time invariant component *i.e.*, mean value.

3. Problem formulation

Denote here as follows: $\boldsymbol{h}$ — filter impulse response, $\boldsymbol{x}$ — filter input signal .

$$\boldsymbol{h} = (h_1, h_2, \dots h_{2n}) , \tag{1}$$

$$\boldsymbol{x} = (x_1, x_2, \dots x_{4n+1}) , \tag{2}$$

where, $n = 1, 2, \dots$.

Now, denote detrended function of x:

$$\boldsymbol{d} = \boldsymbol{x} - T(\boldsymbol{x}) , \tag{3}$$

where $T(\boldsymbol{x})$ is the best L_2 straight-line approximation of x, referred to as the linear trend.

Consequently, consider the convolution

$$\boldsymbol{y} = \boldsymbol{h} * \boldsymbol{d}\,, \tag{4}$$

where asterisk stands for convolution.

Now, define the replica $\boldsymbol{r}$, as follows:

$$\boldsymbol{r} = \mathrm{dif}\left(y_{2*n}, y_{2*n+1} \cdots y_{4*n+1}\right), \tag{5}$$

where dif stands for the first order difference, (numerical derivative).

It is here assumed, that the replica $\boldsymbol{r}$, defined by (5) corresponds to the following fragment $\boldsymbol{s}$ of input detrended signal $\boldsymbol{d}$

$$\boldsymbol{s} = \left(d_{n+1}, \ldots d_{3*n+1}\right). \tag{6}$$

Signal $\boldsymbol{s}$, given by (6), will be referred to as the original.

Now, let us formulate the main thesis of this contribution, as follows:

Thesis:

Assume that for any value of n the impulse response $\boldsymbol{h}$ given by (1) equals the impulse response of the Chebyshev deconvolution filter for odd pair of impulses defined in [3]. Then, the replica $\boldsymbol{r}$, defined by (5) is a very close approximation of the original $\boldsymbol{s}$, defined by (6).

4. Computations

The analytical proof for the thesis formulated in paragraph 3 was not found. However, thru the process of computations this thesis can be confirmed on the grounds of approximation theory, at least to the degree of appropriateness and accuracy of the used norm. To confirm the said thesis the covariance coefficient for the original $\boldsymbol{s}$ and replica $\boldsymbol{r}$, both normalized in L_2, *versus* $2n$ length of filter impulse response was computed.

The database of one minute quotations for the future contracts on WIG 20 index of Warsaw Stock Exchange was used. It covers period of time from October 30, 2001 through June 16, 2003, that is ca. 145000 samples. Data has been divided into segments of the length of $4n + 1$. This way for a given filter length $2n$, a sequence of originals $\boldsymbol{s}$, and corresponding replicas $\boldsymbol{r}$, both of length $2n+1$, were defined. Computations were performed for $n \in [3, 150]$ for the whole database, and the results were averaged.

The graph of the covariance coefficient *versus* n from 3 to 150 is presented in Fig. 1.

In addition to the above some check computations were performed using a number of other, however shorter, data sequences from financial and capital markets, such as Dow Jones Industrial Average (DJIA) index and various currency crosses. In all cases the covariance coefficient varied around approximately 96 percent.

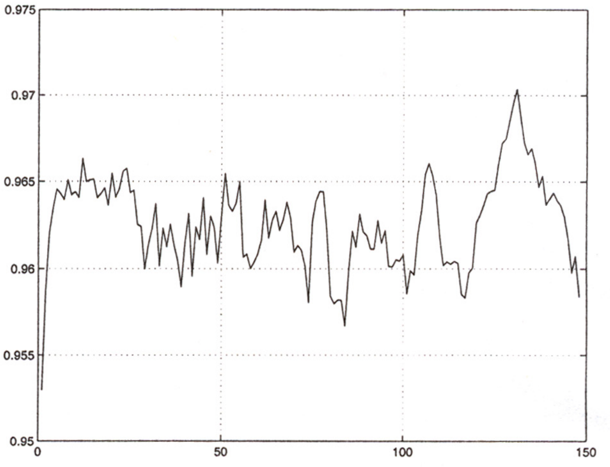

Fig. 1. Covariance coefficient for the original s and replica r.

5. Conclusions

On the grounds of approximation theory the Chebyshev minimax norm related problems, as a rule, do not have analytical solutions. For that reason the most commonly used norm, which always yields analytical, unique solution is L_2. It should be noted that the filter presented here is the best solution to the original deconvolution problem in terms of Chebyshev minimax norm, [3]. Moreover, the impulse response of the filter is given by closed form analytic formulae, which for Chebyshev minimax norm based problems is rather an exception. The aim of this paper is to show that the presented filter can be considered as a very good estimator for the detrended data of any length, thus being an alternative solution to L_2 based solutions. In these of the financial time series analyses, where detrended value of signal is an important figure of merit, the Chebyshev minimax deconvolution filter presented here can be a novel, useful tool. Specifically, this filter can be used for techniques of determining Hurst exponent, which require extraction of the detrended value of input signal, *i.e.* [4].

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